Geometry (input)

$OD := 3.5 \cdot in$ $wall := 0.216 \cdot in$

$OD := 2.375 \cdot in$ $wall := 0.154 \cdot in$ $ca := 0.0591 \cdot in$ $i_i := 1$ $i_o := 1$

Geometry Calcs.

$wt := wall - ca$ $ID := OD - 2 \cdot wt$ $A_p := \frac{\pi}{4} \cdot \left(OD^2 - ID^2\right)$ $I := \frac{\pi}{64} \cdot \left(OD^4 - ID^4\right)$ $Ro := \frac{OD}{2}$ $Z := \frac{I}{Ro}$

$A_f := \frac{\pi}{4} \cdot ID^2$

Set Sustained Stress Indices

$I_i := \textbf{if}\left(0.75 \cdot i_i > 1, 0.75 \cdot i_i, 1\right)$ $I_i = 1$

$I_o := \textbf{if}\left(0.75 \cdot i_o > 1, 0.75 \cdot i_o, 1\right)$ $I_o = 1$

$I_t := 1$ $I_a := 1$

Loads (input) : use node 6040 on 6040-6050

$P := 100 \cdot psi$ $F_{ax} := 53 \cdot lbf$ $M_i := 81.8 \cdot ft \cdot lbf$ $M_o := 3.7 \cdot ft \cdot lbf$ $M_t := -21.6 \cdot ft \cdot lbf$

C2 shows local axial force as positive but, for the From Node, this is compression: $F_{ax} := -F_{ax}$

B31.3 Code Stresses:

320.2 - sustained:

$$S_b := \frac{\sqrt{\left(I_i \cdot M_i\right)^2 + \left(I_o \cdot M_o\right)^2}}{Z} = 2636.8\ psi$$

$$S_t := \frac{I_t \cdot M_t}{2 \cdot Z} = -347.8\ psi$$

$$F_a := F_{ax} + P \cdot A_f = 322\ lbf$$

$$S_a := \frac{I_a \cdot F_a}{A_p} = 473.7\ psi$$

$$S_L := \sqrt{\left(\left|S_a\right| + S_b\right)^2 + \left(2 \cdot S_t\right)^2} = 3187.3\ psi$$

$$\sigma hoop := \frac{P \cdot ID}{2 \cdot wt} = 1151.3\ psi$$

Assemble the stress components required to calculate the three dimensional stress intensity at four locations through the pipe wall.

Calculate the stresses along the pipe diameter which is perpendicular to the resultant bending moment.

1: outside surface, moment causes tension
2: inside surface, moment causes tension
3: inside surface, moment causes compression
4: outside surface, moment causes compression

Intermediate calc's:

$$Ri := \frac{ID}{2} \qquad Ain := A_f \qquad Axs := A_p$$

$$axial := F_a \qquad bend := \sqrt{(i_i \cdot M_i)^2 + (i_o \cdot M_o)^2}$$

$$T := M_t$$

longitudinal stress

hoop stress

radial stress (force this term negative)

shear stress

$$\sigma l := \begin{bmatrix} \frac{axial}{Axs} + \frac{bend \cdot Ro}{I} \\ \frac{axial}{Axs} + \frac{bend \cdot Ri}{I} \\ \frac{axial}{Axs} - \frac{bend \cdot Ri}{I} \\ \frac{axial}{Axs} - \frac{bend \cdot Ro}{I} \end{bmatrix} \qquad \sigma h := \begin{bmatrix} P \cdot \frac{Ri^2}{(Ro^2 - Ri^2)} \cdot \left(\frac{Ro^2}{Ro^2} + 1\right) \\ P \cdot \frac{Ri^2}{(Ro^2 - Ri^2)} \cdot \left(\frac{Ro^2}{Ri^2} + 1\right) \\ P \cdot \frac{Ri^2}{(Ro^2 - Ri^2)} \cdot \left(\frac{Ro^2}{Ri^2} + 1\right) \\ P \cdot \frac{Ri^2}{(Ro^2 - Ri^2)} \cdot \left(\frac{Ro^2}{Ro^2} + 1\right) \end{bmatrix} \qquad \sigma r := \begin{bmatrix} 0 \\ -P \\ -P \\ 0 \end{bmatrix} \qquad \tau := \begin{bmatrix} \frac{T \cdot Ro}{2 \cdot I} \\ \frac{T \cdot Ri}{2 \cdot I} \\ \frac{T \cdot Ri}{2 \cdot I} \\ \frac{T \cdot Ro}{2 \cdot I} \end{bmatrix}$$

Sa and Sb are the principal 2D stresses in the plane normal to the radial direction

$$Sa(p) := \frac{\sigma l_p + \sigma h_p}{2} + \frac{\sqrt{\left(\sigma l_p - \sigma h_p\right)^2 + \left(2 \cdot \tau_p\right)^2}}{2} \qquad Sb(p) := \frac{\sigma l_p + \sigma h_p}{2} - \frac{\sqrt{\left(\sigma l_p - \sigma h_p\right)^2 + \left(2 \cdot \tau_p\right)^2}}{2} \qquad Sc(p) := \sigma r_p$$

$$\sigma l = \begin{bmatrix} 3.1 \cdot 10^3 \\ 2.9 \cdot 10^3 \\ -2 \cdot 10^3 \\ -2.2 \cdot 10^3 \end{bmatrix} psi \qquad \sigma h = \begin{bmatrix} 1103.4 \\ 1203.4 \\ 1203.4 \\ 1103.4 \end{bmatrix} psi \qquad \sigma r = \begin{bmatrix} 0 \\ -100 \\ -100 \\ 0 \end{bmatrix} psi \qquad \tau = \begin{bmatrix} -347.8 \\ -320 \\ -320 \\ -347.8 \end{bmatrix} psi$$

Principal stresses at positions 1 to 4

$$Sq := \begin{bmatrix} Sa(1) & Sa(2) & Sa(3) & Sa(4) \\ Sb(1) & Sb(2) & Sb(3) & Sb(4) \\ Sc(1) & Sc(2) & Sc(3) & Sc(4) \end{bmatrix} \qquad Sq = \begin{bmatrix} 3169 & 2958 & 1236 & 1140 \\ 1045 & 1145 & -1984 & -2200 \\ 0 & -100 & -100 & 0 \end{bmatrix} \; \mathbf{psi}$$

sort stresses at each position (first in ascending order, then in reverse order)

$$Q(p) := \text{sort}\left(Sq^{\langle p \rangle}\right) \qquad S(p) := \text{reverse}(Q(p))$$

$$S(1) = \begin{bmatrix} 3169 \\ 1045 \\ 0 \end{bmatrix} \mathbf{psi} \qquad S(2) = \begin{bmatrix} 2958 \\ 1145 \\ -100 \end{bmatrix} \mathbf{psi} \qquad S(3) = \begin{bmatrix} 1236 \\ -100 \\ -1984 \end{bmatrix} \mathbf{psi} \qquad S(4) = \begin{bmatrix} 1140 \\ 0 \\ -2200 \end{bmatrix} \mathbf{psi}$$

maximum principal stress (S1) at position "p"

$S1(p) := S(p)_1$ $S1(1) = 3169\ \mathbf{psi}$ $S1(2) = 2958\ \mathbf{psi}$ $S1(3) = 1236\ \mathbf{psi}$ $S1(4) = 1140\ \mathbf{psi}$

$S2(p) := S(p)_2$ $S2(1) = 1045\ \mathbf{psi}$ $S2(2) = 1145\ \mathbf{psi}$ $S2(3) = -100\ \mathbf{psi}$ $S2(4) = 0\ \mathbf{psi}$

minimum principal stress (S3) at position "p"

$S3(p) := S(p)_3$ $S3(1) = 0\ \mathbf{psi}$ $S3(2) = -100\ \mathbf{psi}$ $S3(3) = -1984\ \mathbf{psi}$ $S3(4) = -2200\ \mathbf{psi}$

SI:: Maximum Shear Stress Intensity at position "p"

$$SI(p) := S1(p) - S3(p) \qquad \_3DMax := \begin{bmatrix} SI(1) \\ SI(2) \\ SI(3) \\ SI(4) \end{bmatrix} = \begin{bmatrix} 3169.1 \\ 3058.1 \\ 3220 \\ 3339.7 \end{bmatrix} \mathbf{psi}$$

$$MaxStressIntensity := \max(\_3DMax) = 3339.7\ \mathbf{psi}$$

In non-fatigue applications, maximum stress intensity is limited by material yield stress. (Actually, maximum shear stress which is half of the stress intensity is limited by one half of yield.)

Another comparision - von Mises or octahedral shear stress (also known as equivalent stress since this stress calculation is equivalent to the energy of distortion calculation) is limited by yield stress times square root of 2 divided by 3 (.47Sy).

SOct::Octahedral Shear Stress (or Equivalent Stress)

$$SOct(p) := \frac{1}{3} \cdot \sqrt{(S1(p) - S2(p))^2 + (S2(p) - S3(p))^2 + (S3(p) - S1(p))^2}$$

$$OctMax := \begin{bmatrix} SOct(1) \\ SOct(2) \\ SOct(3) \\ SOct(4) \end{bmatrix} \qquad MaxOctShear := \max(OctMax) \qquad MaxOctShear = 1386 \ psi$$

$$MaxOctShear = 1386 \ psi$$

Following illustrates the four positions where stess is calculated: